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Finite-element simulation of temperature-dependent three-point bending process of glass

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Abstract Simulation of strains and stresses distributions in the glass subjected to a bending load during heat treatment is presented in the paper. The main objective of the work is to combine the temperature-dependent experimental test of three-point bending with simulation of this test and to apply inverse analysis to determine the properties of glass. The thermo-mechanical analysis (TMA) was used to test a temperature-dependent glass deformation. The Adina finite element software was used for simulations of the viscous flow. Two parameters in the Williams–Landell–Ferry (WLF) equation were identified by optimization of the square root error between measured and predicted deflection of the sample. Performed experiments and simulations yielded the values of these coefficients: $C_1 = 26.3$ and $C_2 = 62.7$. The proposed model with the optimized coefficients confirmed good agreement with the experimental data.

Keywords Finite-element simulation · Glass · Viscous flow · Three-point bending · Williams–Landell–Ferry equation

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Introduction

The conventional way of glass formation requires cooling of the melt below the glass transition without the occurrence of crystallization. Simultaneously, during the process, a shape of the wares is being obtained by usage of difference methods, e.g., blowing, press-blowing, pressing, etc. For all these methods, the applied force is to induce the viscous flow of the glass. On the other hand, the mechanical strength properties of the ready-made glassy product working near the softening temperature are also dependent on the viscosity flow which is an exponential function of the temperature. The viscosity of the glass can be predicted with reasonable accuracy by the additive law for the most popular glass composition. The knowledge of the viscosity function is indispensable for the appropriate glass formation but it do not resolve a problem of molding, i.e., heat transfer process [1], stretching and thinning of the zones where the temperature locally increases. Thus, the simulation of glass forming is a complex thermo-mechanical problem [2–4].

To approach the problem of the viscous flow of glass being under load the simulation of strains and stresses distributions in the glass sample subjected to a bending load during heat treatment is presented in the paper. The main objective of the work is to combine the temperature-dependent test of three-point bending with simulation and to apply inverse analysis to determination of properties of glass. To reach this objective the constitutive model, which accounts for the properties of glass at scanning temperatures, was implemented into the finite-element code.

Experiment

In order to obtain samples, a peace of commercial tableware glass was cut off to get 6 × 8 mm plates of 0.7, 1.35, and 2.0 mm thickness, respectively. Both parallel surfaces of the plates were polished mechanically.

Thermo-mechanical analysis (TMA) was conducted on Perkin-Elmer TMA-7 with a sapphire three-point bending kit. The applied method is a modification of the standard test method for strength of glass by flexure [5]. For all series of samples three loads of 100, 200, and 400 mN were applied, respectively. The load was constant and the temperature was increased with a rate of 5 °C/min during the experiment.

From the obtained curves of displacement as a function of temperature, the point of softening and the value of viscous strain was calculated for the applied loads.

Model

Rheological material model determines relationship between stress, displacement, strain, and strain rate. In consequence, equation of state is described in the form:

$$f\left(\sigma, \frac{d\sigma}{dt}, \varepsilon, \frac{d\varepsilon}{dt}\right) = 0 \tag{1}$$

where σ stress, ε strain.

The total strain of viscoelastic material is a sum of elastic and viscous parts. Thus, two effects have to be considered: creep and stress relaxation. According to the Maxwell rheological model the effect of the stress relaxation can be determined by the relaxation function $G(t)$:

$$\sigma(t) = G(t)\varepsilon \tag{2}$$

where t time, $G(t)$ relaxation function, which describes changes of shear modulus with the time. The effect of the creep can be described as:

$$\varepsilon(t) = J(t)\sigma \tag{3}$$

where $J(t)$ creep function.

Relation between stress and strain is linear and time dependent, so mechanical behavior of viscoelastic material is expressed in the tensor notation by:

$$\sigma_{ij}(x, t) = \int_{-\infty}^{t} \bar{G}_{ij}(x, t-\tau)\frac{\partial \varepsilon_{kl}}{\partial \tau}(x, \tau)d\tau \tag{4}$$

$$\varepsilon_{ij}(x, t) = \int_{-\infty}^{t} \bar{J}_{ij}(x, t-\tau)\frac{\partial \sigma_{kl}}{\partial \tau}(x, \tau)d\tau \tag{5}$$

where σ_{ij} stress tensor, ε_{ij} strain tensor, $\bar{G}_{ij}$ relaxation function tensor, $\bar{J}_{ij}$ creep function tensor.

The relaxation tensor is divided into the elastic part G_0, which is time independent, and time dependent relaxation function G_{ij}. An assumption is made that for $t < 0$, $\sigma_{ij} = 0$, $\varepsilon_{ij} = 0$, and $\varepsilon_{ij} \neq 0$ for $t = 0$, so Eq. 4 becomes:

$$\sigma_{ij}(x, t) = G_0(x)\varepsilon_{kl}(x, 0) + \int_{0}^{t} G_{ij}(x, t-\tau)\frac{\partial \varepsilon_{kl}}{\partial \tau}(x, \tau)d\tau \tag{6}$$

Equation 5 is written in the following form:

$$\varepsilon_{ij}(x, t) = J_0(x)\sigma_{kl}(x, 0) + \int_{0}^{t} J_{ij}(x, t-\tau)\frac{\partial \sigma_{kl}}{\partial \tau}(x, \tau)d\tau \tag{7}$$

Equations 6 and 7 in varying temperature conditions are given by:

$$\sigma_{ij}(x, t) = G_0(x)\varepsilon_{kl}(x, 0) + \int_{0}^{t} G_{ij}(x, \xi-\zeta)\frac{\partial \varepsilon_{kl}}{\partial \tau}(x, \zeta)d\zeta \tag{8}$$

$$\begin{aligned}\varepsilon_{ij}(x, t) = {} & J_0(x)[\sigma_{kl}(x, 0) - \alpha_0\theta(t)]\sigma_{kl}(x, 0) \\ & + \int_{0}^{t} J_{ij}(x, \xi-\zeta) - \alpha_0\theta(\zeta)\frac{\partial \sigma_{kl}}{\partial \tau}(x, \zeta)d\zeta\end{aligned} \tag{9}$$

where

$$\zeta = \int_{0}^{\tau} a_T[T(\eta)]d\eta, \ \xi = \int_{0}^{\tau} a_T[T(\tau)]d\tau \tag{10}$$

$\theta(t)$ pseudotemperature, α_0 coefficient of thermal expansion, $a_T(T)$ temperature shift factor.

Time dependent part of the relaxation function can be determined in a simple shear test by deformation of the sample and stress measurement at a constant strain. Problem of Maxwell model identification is equivalent to approximation of measurement data by sum of exponentials. Prony method leads to identification scheme, which is simple for implementation. In consequence, the relaxation functions: shear modulus and bulk modulus, are typically modeled with Prony series:

$$G(t) = G_0\left[1 - \sum_{i=1}^{n} p_i \exp\left(-\frac{t}{\tau_i}\right)\right] \tag{11}$$

$$J(t) = J_0\left[1 - \sum_{i=1}^{n} p_i \exp\left(-\frac{t}{\tau_i}\right)\right] \tag{12}$$

where p_i Prony constants, τ_i relaxation time.

The properties of viscoelastic material are time and temperature-dependent. The properties determined at one temperature can be transposed to another using the time–temperature superposition theorem. Williams–Landell–Ferry (WLF) equation gives the temperature shift factor [6, 7]:

$$\log(a_T) = \frac{-C_1(T - T_g)}{C_2 + T - T_g} = \log\frac{\eta}{\eta_0} \quad (13)$$

where C_1, C_2 material constants dependent on T_g, T temperature, T_g reference temperature (glass transition temperature), η viscosity, η_0 viscosity at glass transition.

Mazurin model is an alternative to the model described above. It is not used in the present work, but it is described briefly for comparison. This model is based on the Tool-Narayanaswamy's relaxation theory but it uses different formula for the temperature–structure dependence of the mean relaxation time. In the Mazurin solution the mean relaxation time is simply related to the viscosity. In agreement with the relaxation theory of annealing, a mathematical interpretation was proposed for the physical nature of the decrease in the effective thermal-expansion coefficient of a glass product with an increase in the cooling rate. Mazurin presented a universal method for analysis of stresses that are caused by difference of thermal expansion of materials and that depend on the processes of structural and mechanical relaxation in glass. Mazurin model is based on the non-linear regression analysis. According to [8–10], this model gives erroneous results in varying temperatures.

Finite-element method solution

The numerical model of glass deformation in the test of temperature-dependent three-point bending was created in this work. The program of numerical simulations was convergent with the range of investigations. The Adina software was employed for the analysis of this problem. Viscoelastic material model was used. Figure 1 shows the solution domain and the finite-element mesh created for the glass samples. The mesh was composed of 650 triangular elements.

Values of the glass properties used in this model are shown in Table 1. The shear modulus and bulk modulus are predicted by the Prony series with Eqs. 11 and 12.

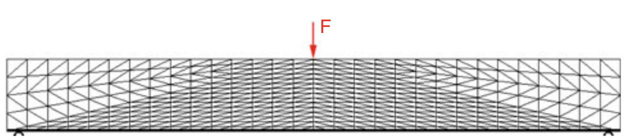

Fig. 1 The solution domain and the finite-element mesh

Table 1 Properties of glass used in the FE model

Density/kg m^{-3}	2497.5
Coefficient of thermal expansion/K^{-1}	92 × 10^{-7}
Reference temperature (transformation temperature)/°C	490

Results

The values of the coefficients C_1 and C_2, which give the best agreement between measurements and predictions, were the objective of the simulations. The idea of inverse analysis of the experimental tests [11] was applied with the vector of

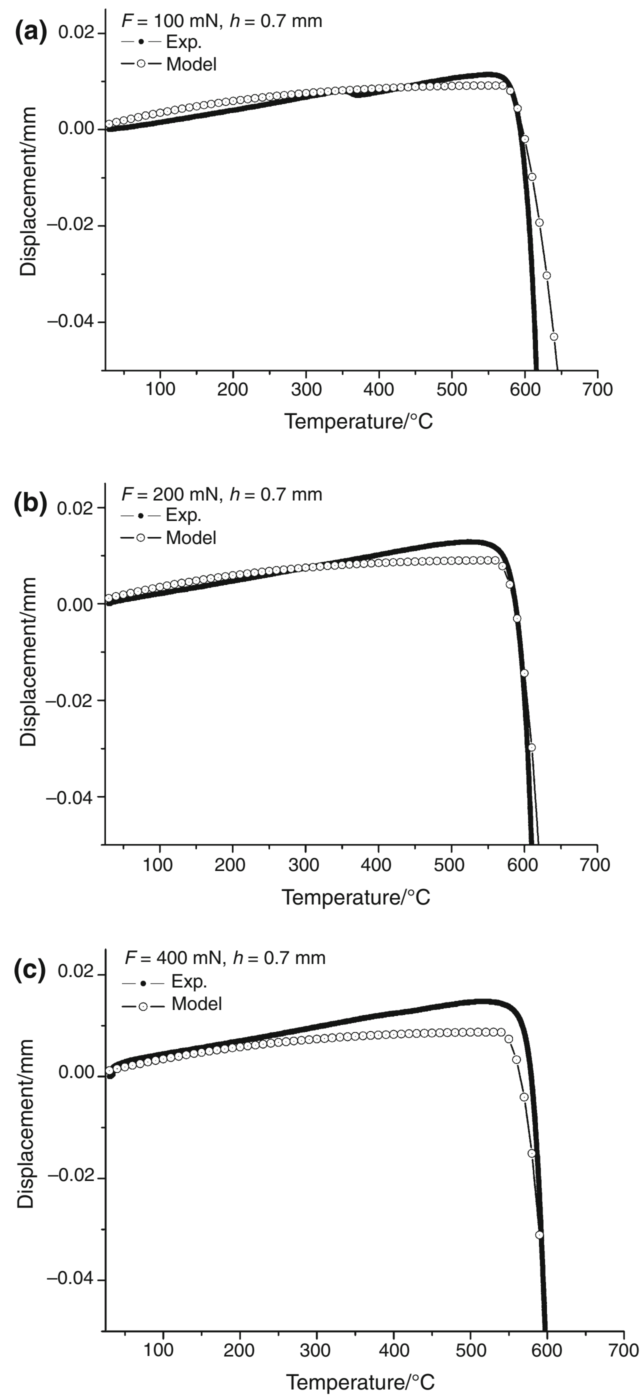

Fig. 2 Results of numerical simulations of deformation of glass (*open points*) compared with the measurements (*filled points*), for the different loads: **a** 100 mN, **b** 200 mN, **c** 400 mN. Thickness of the sample 0.7 mm

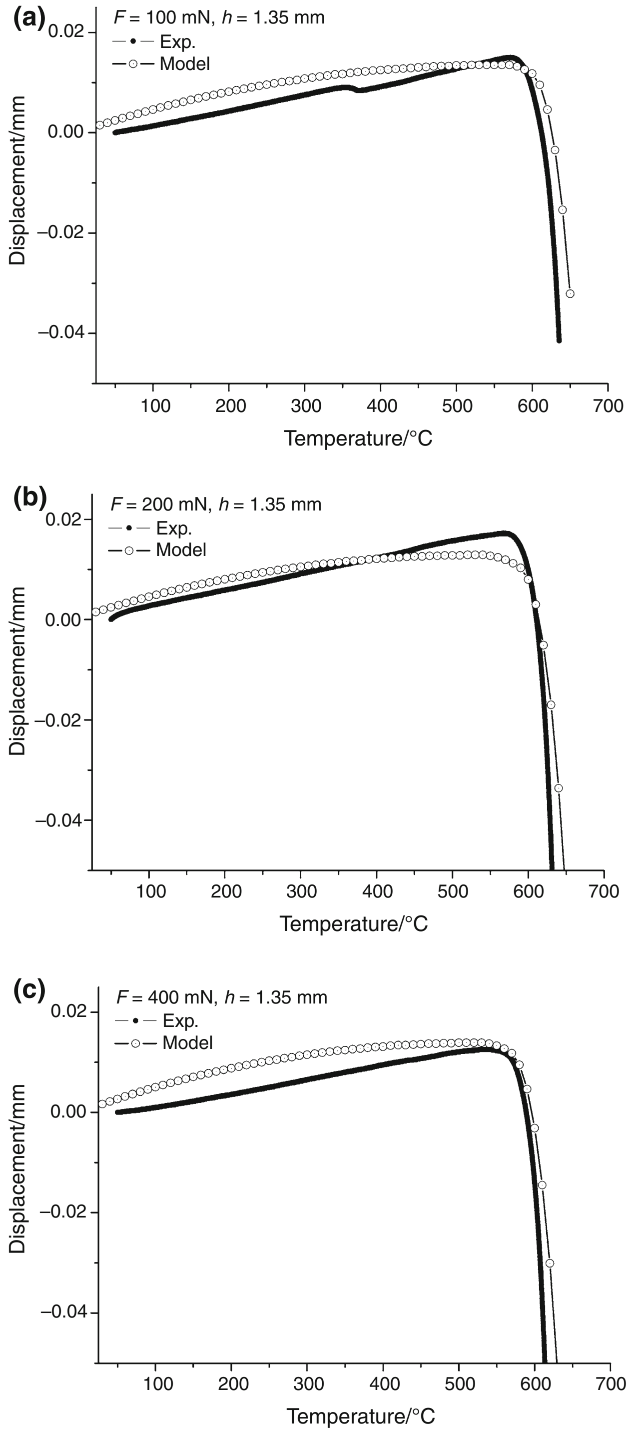

Fig. 3 Results of numerical simulations of deformation of glass (*open points*) compared with the measurements (*filled points*), for the different loads: **a** 100 mN, **b** 200 mN, **c** 400 mN. Thickness of the sample 1.35 mm

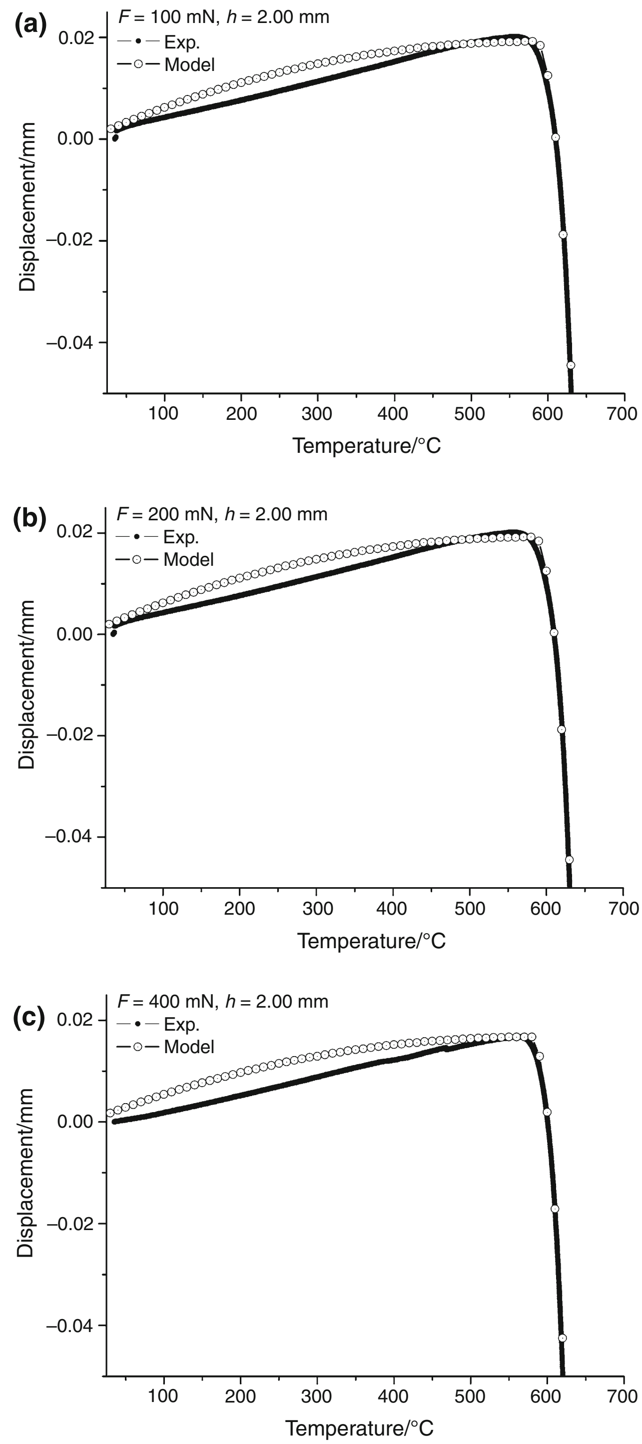

Fig. 4 Results of numerical simulations of deformation of glass (*open points*) compared with the measurements (*filled points*), for the different loads: **a** 100 mN, **b** 200 mN, **c** 400 mN. Thickness of the sample 2.0 mm

optimization variables $\mathbf{p} = \{C_1, C_2\}^T$. The objective function was formulated as the square root error between measured and calculated location of the top central point of the sample (the point, at which the load is applied):

$$\Phi = \sqrt{\frac{1}{N_t}\sum_{i=1}^{N_t}\frac{1}{N_s}\sum_{j=1}^{N_s}\left[\frac{y_{\mathrm{mij}}(T) - y_{\mathrm{cij}}(\mathbf{p}, T)}{y_{\mathrm{mij}}(T)}\right]^2} \tag{14}$$

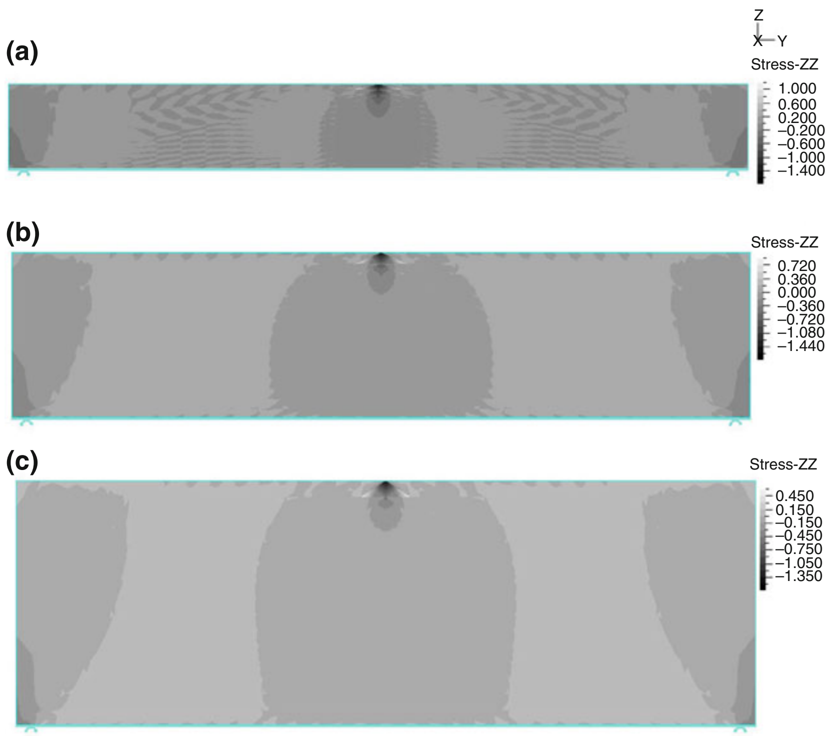

Fig. 5 Calculated distribution of stress component σ_{zz} for load 100 mN at the cross-section of the sample in thickness: **a** 0.7 mm, **b** 1.35 mm, **c** 2.00 mm at the temperature of the softening initiation

where y_{mij}, y_{cij} measured and calculated location of the considered point, respectively, N_t number of performed tests, N_s number of sampling points in one test.

Minimum of the objective function (14) was obtained for $C_1 = 26.3$ and $C_2 = 62.7$.

In order to validate the model, simulations of all tests using the optimum coefficients C_1 and C_2 were performed. Results of the numerical simulations of viscous flow performed for various thickness of samples (0.7, 1.35, 2.0 mm) and three loads (100, 200, 400 mN) were compared with the experimental data (Figs. 2, 3, and 4).

Figures show the TMA curves with an effect of the thermal expansion in the first stage of heating and process of subsiding at the range of lower viscosity. The softening temperature observed as the point of maximum on TMA curves is depended on the applied load and thickness of the glassy sample. With an increasing of the load and decreasing of the thickness, the softening temperature shifts to the lower temperature. Moreover, the parameters have significant influence on the slop of TMA curve in a range of the viscous flow.

Predictive capabilities of the developed model are demonstrated in Fig. 5, where selected examples of the stress distributions at the cross-section of the sample are presented. A stress component in the vertical direction (σ_{zz}) at the stage of initiation of viscous flow of glass for 100 mN load is compared for the 0.7, 1.35, and 2.00 mm thick samples. The stresses are compressive above the supports and under the point where the load is applied. Tensile stresses occur in the remaining part of the sample.

Conclusions

Identification of properties of glass was performed on the basis of temperature-dependent three-point bending test and FE simulations of this test. Two parameters in the WLF equation were the optimization variables. Performed experiments and simulations yielded the values of these coefficients: $C_1 = 26.3$ and $C_2 = 62.7$.

Validation of the model with the optimized coefficients confirmed good agreement with the experimental data. Some discrepancies observed for the thinner samples should be associated with the assumption of the 2D model and the boundary condition effect.

The proposed model of glass flow behavior can be applied to industrial processes after usage of three dimensional simulation.

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